(3) **Tangent ∥ to axis** $\frac{dy}{d\theta} = 2\cos\theta, \quad \frac{dx}{d\theta} = -2\sin 2\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cancel{2}\cos\theta}{-\cancel{2}\sin 2\theta} = -\frac{\cancel{\cos\theta}}{2\sin\theta\cancel{\cos\theta}} = -\frac{1}{2\sin\theta}$$

Tangent ∥ to x-axis: put $\frac{dy}{dx} = 0 \Rightarrow \frac{1}{2\sin\theta} = 0$

$\Rightarrow 1 = 0$

Hence there is no tangent ∥ to x-axis.

Tangent ∥ to y-axis: put $\frac{dy}{dx} = \infty \Rightarrow \frac{1}{2\sin\theta} = \infty$

$\Rightarrow \sin\theta = 0$

$\Rightarrow \theta = n\pi$

$x = \cos 2n\pi, \quad y = 2\sin n\pi$

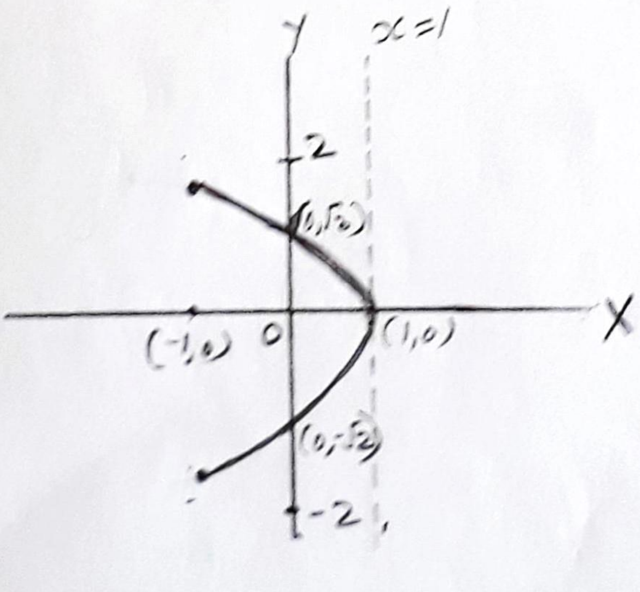
$x = 1, \quad y = 0$

∴ Tangent ∥ to y-axis is at (1, 0).

(4) **Asymptotes:**

Asymptote ∥ to axis: Here we can not find any limiting value of θ for which one variable is finite and other is infinite. So there is no possibility of parallel asymptotes.

Oblique asymptotes: There does not exists value of θ for which both $x \to \infty$ and $y \to \infty$. So oblique asymptote does not exists.



Oblique Asymptote: If $t \to 0$, then $x \to \infty$, $y \to \infty$. So there is a possibility of oblique asymptotes. Oblique asymptote is given by eq. $y = mx + c$.

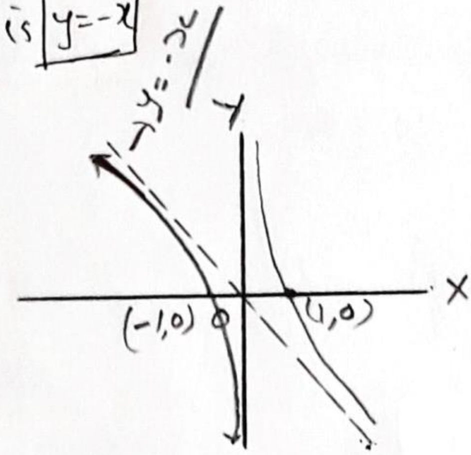
where $m = \lim_{t \to 0} \frac{dy}{dx} = \lim_{t \to 0} -(t^2+1)$

$\Rightarrow m = -1$

Also, $c = \lim_{t \to 0} (y - mx) = \lim_{t \to 0} (y + x) = \lim_{t \to 0} \left[t - \frac{1}{t} + \frac{1}{t} \right] = \lim_{t \to 0} t = 0$

$c = 0$

$\therefore$ Oblique Asymptote is $\boxed{y = -x}$



Q.5 Sketch the curve given by

$x = \cos 2\theta, \ y = 2 \sin \theta$

Solution:

(1) Intercept:

x-intercept: put $y = 0$, we get,

$2 \sin \theta = 0 \Rightarrow \sin \theta = 0 \Rightarrow \theta = n\pi$

$x = \cos 2n\pi \Rightarrow x = 1$

$x = 1$ is the x-intercept

y-intercept: put $x = 0$, we get

$\cos 2\theta = 0 \Rightarrow 2\theta = (2n+1)\pi/2 \Rightarrow \theta = (2n+1)\frac{\pi}{4}$

$y = 2 \sin (2n+1)\pi/4 \Rightarrow \boxed{y = \pm \sqrt{2}}$

So $y = \pm\sqrt{2}$ is the y-intercept

(2) Extent:

$-1 \le \cos \theta \le 1$	$-1 \le \sin \theta \le 1$
$-1 \le \cos 2\theta \le 1$	$-2 \le 2 \sin \theta \le 2$
$-1 \le x \le 1$	$-2 \le y \le 2$

Q.4 Sketch the curve given by

$$x = \frac{1}{t}, \quad y = t - \frac{1}{t}$$

Solution:

(1) Intercepts:

X-intercept put $\cancel{y} = 0$, we get

$$t - \frac{1}{t} = 0 \Rightarrow \frac{t^2-1}{t} = 0 \Rightarrow t^2 - 1 = 0 \Rightarrow t = \pm 1$$

$\because x = \frac{1}{t} \Rightarrow x = \pm 1$, ~~are~~

$\therefore x = \pm 1$ are x-intercepts

Y-intercept put $x = 0$, we get

$\frac{1}{t} = 0 \Rightarrow 0 = 1$, which is not possible.

so there is no Y-intercept.

(2) Extent: $t \in R$ $\Rightarrow x \in R$ | $t \in R$ $y \in R$

(3) Tangent parallel to axis: $\frac{dx}{dt} = -\frac{1}{t^2}$, $\frac{dy}{dt} = 1 + \frac{1}{t^2} = \frac{t^2+1}{t^2}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{t^2+1}{t^2}}{-\frac{1}{t^2}} = -(t^2+1).$$

Tangent ll to X-axis: put $\frac{dy}{dx} = 0$, we get

$t^2 + 1 = 0 \Rightarrow t^2 = -1$.

$\therefore$ Tangent ll to X-axis does not exists.

Tangent ll to Y-axis $\frac{dy}{dx} \to \infty \Rightarrow (t^2+1) \to \infty$

Tangent ll to y-axis does not exists.

(4) Asymptotes

Asymptote ll to axis If $t \to \infty$, then $x \to 0$ and $y \to \infty$. so

$x = 0$ is asymptote parallel to y-axis.

Intervals are $(-\infty,-1), (-1,0), (0,1), (1,\infty)$ $-1 \quad 0 \quad +1$

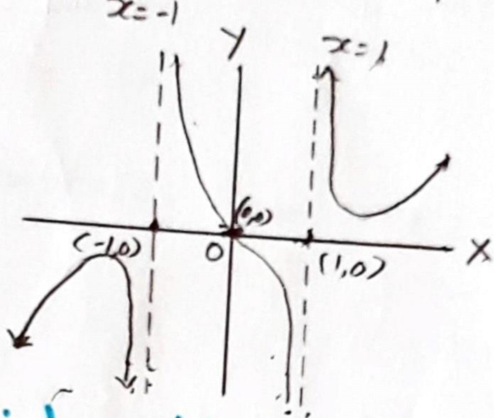
$$y = \frac{x^3}{x^2-1} = \frac{x^3}{(x+1)(x-1)}$$

$(-\infty,-1)$ $y = \frac{(-)}{+} = - < 0 \Rightarrow y < 0$

$(-1,0)$ $y = \frac{-}{(+)(-)} = \frac{(-)}{(-)} = + \Rightarrow y > 0$

$(0,1)$ $y = \frac{+}{(+)(-)} = \frac{(+)}{(-)} = - \Rightarrow y < 0$

$(1,\infty)$ $y = \frac{+}{(+)(+)} = \frac{+}{+} = +$
$\Rightarrow y > 0$



Q.3. Discuss the intercepts, symmetry, horizontal and vertical asymptotes for

$$y = \frac{x^2-9}{x^2-4}$$

Also sketch the curve.

Sol.

(1) Intercepts:

X-intercept: put $y=0$, we get, $0 = \frac{x^2-9}{x^2-4} \Rightarrow x^2-9=0$
$\Rightarrow x = \pm 3$

Hence $x = \pm 3$ is the x-intercept.

Y-intercept: put $x=0$, we get, $y = \frac{0-9}{0-4} \Rightarrow y = \frac{9}{4}$.

Hence $y = \frac{9}{4}$ is the y-intercept.

(2) Symmetry:

Symmetry about X-axis: Replacing y by $-y$, we get

$$-y = \frac{x^2-9}{x^2-4} \Rightarrow y = -\frac{x^2-9}{x^2-4}$$

Eq. is changed
Curve is not symmetric about x-axis.

Symmetry about Y-axis: Replacing x by $-x$, we get

$$y = \frac{(-x)^2-9}{(-x)^2-4} \Rightarrow y = \frac{x^2-9}{x^2-4}$$

Eq. is not changed
Hence curve is symmetric about y-axis.

Q.2. Discuss the intercepts, symmetry, horizonal and vertical asymptotes for

$$y = \frac{x^3}{x^2-1}.$$

Also sketch the curve.

Sol. (1) **Intercept:**

X-intercept: put $y=0$, we get, $0 = \frac{x^3}{x^2-1}$

$\Rightarrow x^3 = 0$

so $x=0$ is the x-intercept

Y-intercept: put $x=0$, we get

$y = \frac{0}{0-1} \Rightarrow y=0$

so, $y=0$ is the y-intercept.

(2) **Symmetry:**

Symmetry about x-axis: Replacing y by $-y$, we get

$-y = \frac{x^3}{x^2-1} \Rightarrow y = -\frac{x^3}{x^2-1}$

Eq is changed.

Hence curve is not symmetric about x-axis

Symmetry about y-axis: Replacing x by $-x$, we get

$y = \frac{(-x)^3}{(-x)^2-1} \Rightarrow y = \frac{-x^3}{x^2-1}$

Eq is changed

Hence curve is not symmetric about y-axis.

Symmetry about origin: Replacing x by $-x$ and y by $-y$, we get

$-y = \frac{(-x)^3}{(-x)^2-1} \Rightarrow -y = \frac{-x^3}{x^2-1} \Rightarrow y = \frac{x^3}{x^2-1}$

Eq. is unchanged

Hence, Curve is symmetric about origin.

(3) **Asymptotes:**

Vertical Asymptote: Take $x^2-1=0$

$\Rightarrow x = \pm 1$

So $x = \pm 1$ are vertical Asymptotes

Horizontal Asymptote: Comparing $y = \frac{x^3}{x^2-1}$ with $y = \frac{a_0 + a_1x + \cdots + a_m x^m}{b_0 + b_1 x + \cdots + b_n x^n}$

we get $m=3$, $n=2$

$3 > 2 \Rightarrow m > n$

Horizontal Asymptotes are not possible.

(3) Asymptotes :

Vertical Asymptotes Take $x^2 - 9 = 0$.

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

So $x = \pm 3$ are vertical Asymptotes.

Horizontal asymptotes comparing $y = \dfrac{x-1}{x^2-9}$ with

$$y = \frac{a_0 + a_1 x + \cdots + a_n x^n}{b_0 + b_1 x + \cdots + b_m x^m}, \text{ we get}$$

$n = 1$, $m = 2$

Since $2 > 1 \Rightarrow m > n$

then $y = 0$ is Horizontal asymptote.

(4) Sign of function :

Let $x - 1 = 0$ | $x^2 - 9 = 0$

$\Rightarrow x = 1$ | $\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

intervals are $-3 \quad 1 \quad +3$

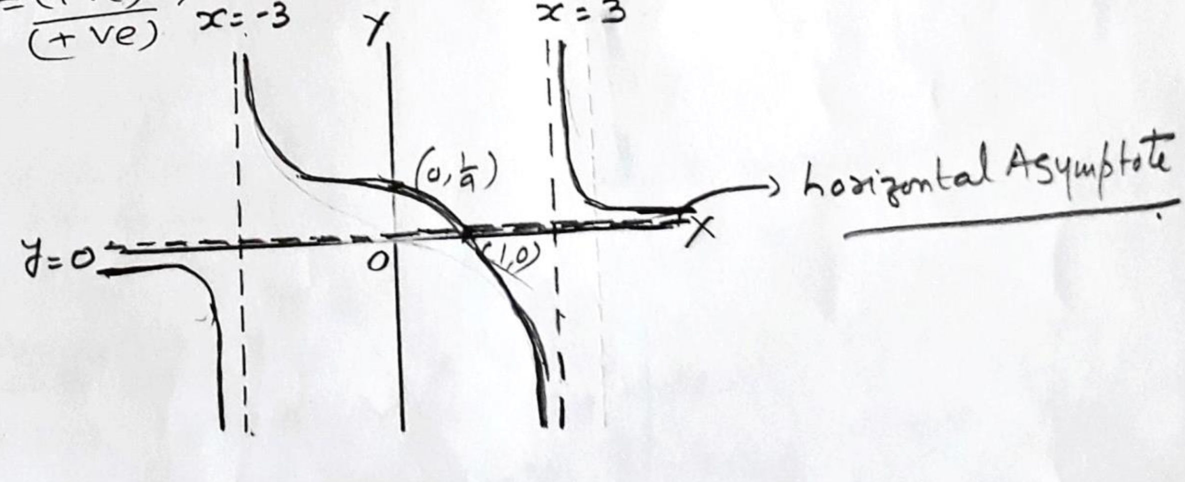
$(-\infty, -3)$, $(-3, 1)$, $(1, 3)$, $(3, \infty)$

$(-\infty, -3)$, $y = \dfrac{(-ve)}{(+ve)} < 0 \Rightarrow y < 0$

$(-3, 1)$, $y = \dfrac{(-ve)}{(-ve)} > 0 \Rightarrow y > 0$

$(1, 3)$, $y = \dfrac{(+ve)}{(-ve)} < 0 \Rightarrow y < 0$

$(3, \infty)$, $y = \dfrac{(+ve)}{(+ve)} > 0 \Rightarrow y > 0$



CHAPTER # 03

Sketching of cartesian Curves and Parameteric Curves

Q.1 Discuss the intercepts, symmetry, horizontal and vertical asymptotes for

$$y = \frac{x-1}{x^2-9}.$$

Hence sketch the curve.

Sol (1) **Intercept :**

X-intercept : Put $y=0$, we get $0 = \frac{x-1}{x^2-9}$

$\Rightarrow x-1 = 0\times(x^2-9) = 0$

$\Rightarrow x = 1$

$\therefore x = 1$, is the x-intercept.

y-intercept : Put $x=0$, we get

$$y = \frac{0-1}{0-9} = \frac{1}{9}$$

So $y = \frac{1}{9}$ is the y-intercept.

(2) **Symmetry :**

Symmetry about X-axis : Replace y by $-y$, we get

$$-y = \frac{x-1}{x^2-9} \Rightarrow y = -\frac{x-1}{x^2-9}$$

Eq is changed.
So curve is not symmetric about x-axis.

Symmetry about Y-axis : Replace x by $-x$, we get

$$y = \frac{-x-1}{(-x)^2-9} = \frac{-x-1}{x^2-9}$$

Equation is changed, so not symmetric about Y-axis

symmetry about origin : Replace x by $-x$ and y by $-y$, we get

$$-y = \frac{-x-1}{(-x)^2-9} = \frac{-x-1}{x^2-9}$$

$$\Rightarrow y = \frac{x+1}{x^2-9}$$

Equation is changed
$\therefore$ Not symmetric about origin.